

**ACTIVE CONTROL OF TRANSIENT TORSIONAL VIBRATIONS DUE TO RUN-UP OF A ROTOR MACHINE DRIVEN BY THE ELECTRIC MOTOR**

Tomasz SZOLC, Łukasz JANKOWSKI

Institute of Fundamental Technological Research, Polish Academy of Sciences

ul. Świętokrzyska 21, 00-049 Warszawa

phone: +48/22/8261281, e-mail: [tszolc@ippt.gov.pl](mailto:tszolc@ippt.gov.pl), [ljank@ippt.gov.pl](mailto:ljank@ippt.gov.pl)

**Summary**

In the paper active control of transient torsional vibrations induced by the electric motor during run-ups of the radial compressor drive system is performed by means of couplings with the magneto-rheological fluid. The main purpose of these studies is a minimisation of vibration amplitudes in order to increase the fatigue durability of the most responsible elements. The theoretical investigations are based on a hybrid structural model of the vibrating mechanical system and sensitivity analysis of the response with respect to the damping characteristics of the control couplings.

**Keywords:** active control, transient vibrations, drive system, electric motor

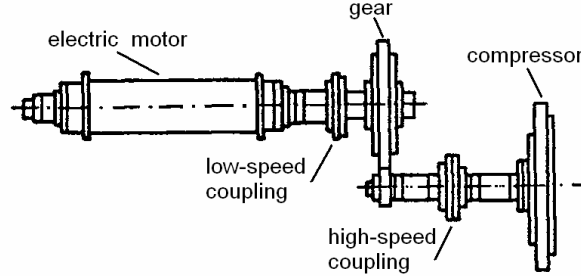
**Introduction**

Transient torsional vibrations due to start-ups of drive trains driven by electric motors are very dangerous for material fatigue of the most heavily affected and responsible elements of these mechanical systems. Thus, this problem has been considered for many years by many authors, e.g. by [1-3]. But till present majority of these studies reduced to standard transient vibration analyses taking into consideration additional dynamic effects caused by elastic couplings, dry friction in clutches, properties of the gear stage meshings, e.g. backlash, electro-mechanical couplings and others.

Active control of torsional vibrations occurring in the drive systems could effectively minimise material fatigue and in this way it would enable us to increase their operational reliability and durability, e.g. in the form of greater number of admissible safe run-ups and run-outs. Unfortunately, one can find not so many published results of research in this field, beyond some attempts performed by active control of shaft torsional vibrations using piezo-electric actuators, [4]. But in such cases relatively small values of control torques can be generated and thus an application of the piezo-electric actuators has to be reduced to low-power drive systems. In this paper active control is going to be realised by couplings with the magneto-rheological fluid (MRF) applied to attenuate transient torsional vibrations excited during start-ups of the large radial compressor drive system driven by the asynchronous motor.

**1. Assumptions for the simulation model and formulation of the problem**

In the considered drive system of the radial compressor power is transmitted from the asynchronous motor to the impeller by means of the low-speed and high-speed rigid couplings, multiplication single-stage gear and shaft segments, as shown in Fig. 1.



**Fig. 1** Mechanical model of the compressor drive system

In order to perform the theoretical investigation of the active control concept applied for this mechanical system a reliable and computationally efficient simulation model is required. In this paper dynamic investigations of the entire drive system are performed by means of the one-dimensional hybrid structural model consisting of continuous visco-elastic macro-elements and rigid bodies. This model is employed here for eigenvalue analyses as well as for numerical simulations of torsional vibrations of the drive train. In this model successive cylindrical segments of the stepped rotor-shaft are substituted by torsionally deformable cylindrical macro-elements of continuously distributed inertial-visco-elastic properties. Since in the drive system of the real compressor the electric motor coils and gears are attached along some rotor-shaft segments by means of shrink-fit connections, the entire inertia of such fragments is increased, whereas usually the shaft cross-sections only are affected by elastic deformations due to transmitted loadings. Thus, the corresponding visco-elastic macro-elements in the hybrid model must be characterized by the geometric cross-sectional polar moments of inertia  $J_{Ei}$  responsible for their elastic and inertial properties as well as by the separate layers of the polar moments of inertia  $J_{Ii}$  responsible for their inertial properties only,  $i=1,2,\dots,n$ , where  $n$  is the total number of macro-elements in the considered hybrid model. Moreover, on the actual operation temperature  $T_i$  can depend values of Kirchhoff's moduli  $G_i$  of the rotor-shaft material of density  $\rho$  for each  $i$ -th macro-element representing given rotor-shaft segment. In the proposed hybrid model of the compressor drive system with a reasonable accuracy for practical purposes inertias of the impeller and gears can be represented by rigid bodies attached to the appropriate macro-element extreme cross-sections.

Torsional motion of cross-sections of each visco-elastic macro-element is governed by the hyperbolic partial differential equations of the wave type

$$G_i(T_i)J_{Ei} \left( 1 + \tau \frac{\partial}{\partial t} \right) \frac{\partial^2 \theta_i(x,t)}{\partial x^2} - \rho (J_{Ei} + J_{Ii}) \frac{\partial^2 \theta_i(x,t)}{\partial t^2} = q_i(x,t), \quad (1)$$

where  $\theta_i(x,t)$  is the angular displacement with respect to the shaft rotation with the average angular velocity  $\Omega$  and  $\tau$  denotes the retardation time in the Voigt model of material damping. The external active, passive and control torques are continuously distributed along the respective macro-elements of the lengths  $l_i$ . These torques are

described by the two-argument function  $q_i(x,t)$ , where  $x$  is the spatial co-ordinate and  $t$  denotes time.

Mutual connections of the successive macro-elements creating the stepped shaft as well as their interactions with the rigid bodies are described by equations of boundary conditions. These equations contain geometrical conditions of conformity for rotational displacements of the extreme cross sections for  $x=L_i=l_1+l_2+\dots+l_{i-1}$  of the adjacent  $(i-1)$ -th and the  $i$ -th elastic macro-elements:

$$\theta_{i-1}(x,t) = \theta_i(x,t) \quad \text{for } x = L_i. \quad (2a)$$

The second group of boundary conditions are dynamic ones, which contain linear equations of equilibrium for external torques  $M_i(t)$  as well as for inertial, elastic and external damping moments. For example, the dynamic boundary condition describing a simple connection of the mentioned adjacent  $(i-1)$ -th and the  $i$ -th elastic macro-elements has the following form:

$$\begin{aligned} M_i(t) - I_{0i} \frac{\partial^2 \theta_i}{\partial t^2} - D_i \frac{\partial \theta_i}{\partial t} - G_{i-1}(T_{i-1}) J_{E,i-1} \left( 1 + \tau \frac{\partial}{\partial t} \right) \frac{\partial \theta_{i-1}}{\partial x} + \\ + G_i(T_i) J_{Ei} \left( 1 + \tau \frac{\partial}{\partial t} \right) \frac{\partial \theta_i}{\partial x} = 0 \quad \text{for } x = L_i, \quad i = 2, 3, \dots, n, \end{aligned} \quad (2b)$$

where  $D_i$  are the external damping coefficients and  $I_{0i}$  denotes the mass polar moment of inertia of the rigid body.

In order to perform an analysis of natural elastic vibrations, all the forcing and viscous terms in the motion equations (1) and boundary conditions (2b) have been omitted. An application of the solution of variable separation for Eqs. (1) leads to the following characteristic equation for the considered eigenvalue problem:

$$\mathbf{C}(\omega) \cdot \mathbf{D} = \mathbf{0}, \quad (3)$$

where  $\mathbf{C}(\omega)$  is the real characteristic matrix and  $\mathbf{D}$  denotes the vector of unknown constant coefficients in the analytical local eigenfunctions of each  $i$ -th macroelement. Thus, the determination of natural frequencies reduces to the search for values of  $\omega$ , for which the characteristic determinant of matrix  $\mathbf{C}$  is equal to zero. Then, the torsional eigenmode functions are obtained by solving equation (3).

The solution for forced vibration analysis has been obtained using the analytical - computational approach. Solving the differential eigenvalue problem (1)-(3) and an application of the Fourier solution in the form of series in the orthogonal eigenfunctions lead to the set of uncoupled modal equations for time coordinates  $\xi_m(t)$ :

$$\ddot{\xi}_m(t) + \tau \omega_m^2 \dot{\xi}_m(t) + \omega_m^2 \xi_m(t) = Q_m(t), \quad m = 1, 2, \dots, \quad (4)$$

where  $\omega_m$  are the successive natural frequencies of the drive system and  $Q_m(t)$  denote the modal external excitations. Although each Eq. (4) has its analytical solution, it can be also solved numerically using a direct integration in order to obtain transient torsional response for the passive and actively controlled system.

## 2. Control of the transient torsional vibrations

The magneto-rheological fluids are functional fluids, whose effective viscosity depends on externally provided magnetic field. This feature makes them perfectly suitable for large couplings with controllable damping characteristic. Besides the ability to generate large damping torques, an important advantage of the MRF-based devices is a low power consumption. External power is needed to supply the electromagnetic coils only, i.e. to modify the dynamic characteristics of the mechanical system, which is the distinguishing feature of the semi-active control. Moreover, the semi-active damping-based approach eliminates a risk of causing instability, which is intrinsically related to the active control paradigm and can occur in the case of an electrical failure, control time delays or in the case of an inaccurate modelling.

Assume there are  $N$  controllable couplings, each with the damping coefficient  $c_j k(t)$ ,  $j=1,2,\dots,N$ , where  $k(t)$  is the collective control variable and  $c_j$  are the coupling-specific multipliers. Each coupling generates the damping torque

$$M_j^D(t) = -c_j k(t) \varpi(x_j, t) = -c_j k(t) \left[ \Omega(t) + \sum_{m=1}^{\infty} \dot{\theta}_m(x_j, t) \right], \quad j = 1, 2, \dots, N, \quad (5)$$

where  $x_j$  is the location of the  $j$ -th coupling. The damping torques  $M_j^D(t)$  modify Eqs. (4) by coupling them with each other and with the equation of the rigid body shaft motion. However, by proper determination of the multipliers  $c_j$ , the most resonant mode can be decoupled from the influence of the average angular velocity  $\Omega(t)$ .

The optimum open-loop control  $k(t)$  can be determined with respect to the two following objectives:

- (1) Maximization of the effectiveness of the damping, which is quantified here as the mean square torque above a given safe level. In practice, this objective can be related to the most resonant eigenvibration mode:

$$F_1[k(t)] = \int \max\left(0, |\xi_r(t)| - \xi_{r(\text{safe})}\right)^2 dt. \quad (6)$$

- (2) Minimization of the energy dissipated due to the damping

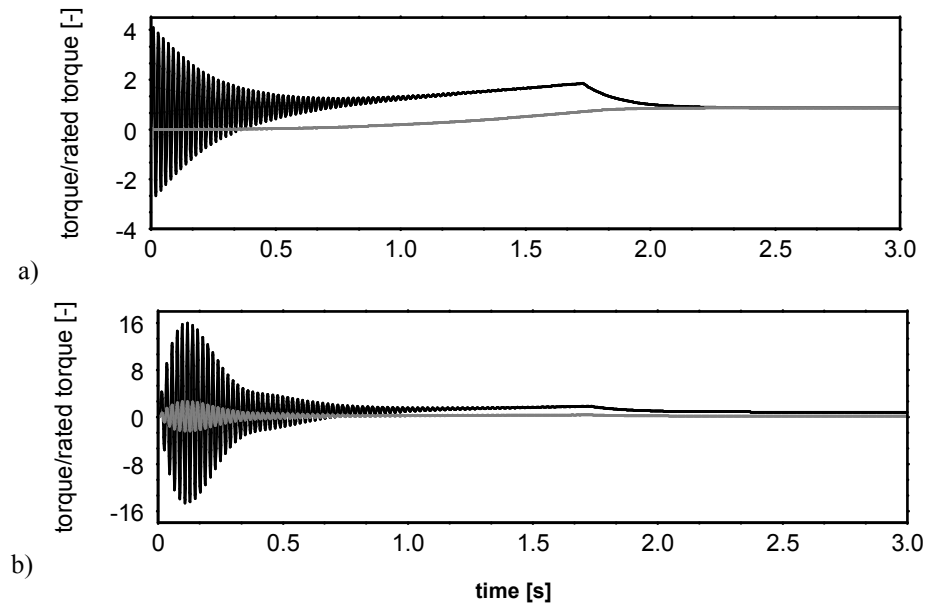
$$F_2[k(t)] = \sum_{j=1}^N \int M_j^D(t) \varpi(x_j, t) dt = \sum_{j=1}^N \int c_j k(t) \left[ \Omega(t) + \sum_{m=1}^{\infty} \dot{\theta}_m(x_j, t) \right]^2 dt. \quad (7)$$

Provided the time history of the control variable  $k(t)$  depends on a finite number of parameters, the compound objective function, composed of the weighted objectives (5) and (6), can be minimized using standard numerical approaches. In this paper it is assumed that  $k(t)$  is a linear combination of a finite set of base functions,  $k(t) = \sum_{i=1}^K \hat{k}_i k_i(t)$ , which makes it dependent on  $K$  parameters  $\hat{k}_i$  and thus suitable for a constrained numerical optimization. A proper choice of the constraints and of the base functions allows also the MRF-specific damping rise and decay rate restrictions to be satisfied. Note that in this way the optimum open-loop control is obtained, which can serve as the best-case reference for possible closed-loop control laws. Moreover, in

the case of the asynchronous motor, as the computational example illustrates below, the optimum control is a simple hold-and-release strategy, which can be easily realized using an open-loop control.

### 3. Computational example and conclusion

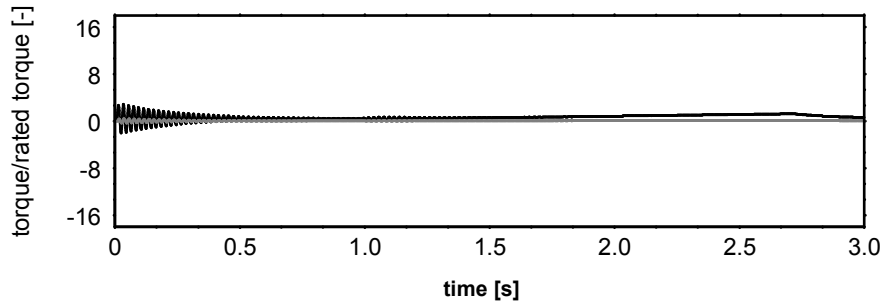
In the computational example the start-up of the large radial compressor driven by the 5 MW asynchronous motor is investigated. This system presented schematically in Fig. 1 is accelerated from a standstill to the nominal operating conditions characterized by the rated retarding torque 31831 Nm at the constant rotational speed 1500 rpm. The values of these quantities are reduced to the motor shaft, where the impeller rotational speed is 4.932 times multiplied by the gear stage. The electromagnetic torque generated by the asynchronous motor has been ‘a priori’ assumed according to the respective relations contained in [5]. The retarding aerodynamic torque produced by the compressor is described by the parabolic function as proportional to the square of the impeller angular velocity. The time history plots of these torques during run-up are illustrated in Fig. 2a by the black and grey lines, respectively.



**Fig. 2** Time histories of the motor and retarding torques (a) and elastic torques (b)

The passive system transient dynamic response due to the start-up is presented in Fig. 2b in the form of time histories of elastic torques transmitted by the shafts in the vicinity of the low-speed coupling (black line) and the high-speed coupling (grey line). From these plots it follows that the transient component of the asynchronous motor torque in the form of attenuated sinusoid of the network frequency 50 Hz and initial amplitude ca. 3.5 times greater than the rated torque value, Fig. 2a, induces very severe

resonance with the first system eigenvibration mode of frequency 47.4 Hz. The maximum amplitudes of the most heavily affected shaft close to the low-speed coupling are almost 16 times greater than the rated torque, Fig. 2b, which is very dangerous for its fatigue durability. Thus, active control of transient torsional vibrations occurring in this drive system during start-ups is very required.



**Fig. 3** Time histories of the elastic torques in the actively controlled system

In the same way as above, in Fig. 3 there are depicted plots of the analogous time histories of the elastic torques transmitted by the mentioned shafts and excited due to run-up of the actively controlled system. From these plots it follows that the corresponding extreme values of the elastic torques have been minimised more than 5 times in comparison with these in Fig. 2b. Here, these reduced amplitudes do not exceed dangerously the transmitted nominal torsional moment, where the exact reduction ratio depends on the technological bounds imposed on the control damping coefficient. Nevertheless, it is to remark that the computed optimum active control is a simple hold-and-release strategy and it results in some breaking of the rigid body motion of the system, which leads to a slight run-up retardation in time.

## Literature

1. B. F. Evans, A. J. Smalley, H. R. Simmons, *Startup of synchronous motor drive trains: the application of transient torsional analysis of cumulative fatigue assessment*, ASME Paper 85-DET-122 (1985).
2. T. Iwatsubo, Y. Yamamoto, R. Kawai, *Start-up torsional vibration of rotating machine driven by synchronous motor*, Proc. of the International Conference on Rotordynamics, IFToMM, Tokyo 1986, pp. 319-324.
3. P. De Choudhury, *Torsional system design relative to synchronous motor start-up with a variable frequency power supply system*, Proc. of the Int. Conference on Rotordynamics, IFToMM, Tokyo 1986, pp. 325-328.
4. P. M. Przybyłowicz, *Torsional vibration control by active piezoelectric system*, *J. of Theoretical and Applied Mechanics*, 33(4) 1995, pp. 809-823.
5. A. Laschet, *Simulation von Antriebssystemen*, Springer-Verlag, Berlin, London, New-York, Paris, Tokyo 1988.